PLUME RISE ABOVE A STACK

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Measurements have been made on the hydrodynamic and thermal components of plume rise above stacks; working formulas are derived.

The plume rise over a stack substantially influences ground-level pollutant concentrations; there have been many papers on this topic [1-7], but not all factors have been fully elucidated. This applies particularly to the plume shape, the maximum height reached, and the effects on the rise from plume parameters, wind parameters, etc. Further, although there is substantial agreement on the hydrodynamic component of the rise, particularly in relation to wind and plume parameters, there is no such agreement for the thermal component.

The thermal component of the plume rise above a stack is [8] given by

$$\Delta h = B \frac{w_0^{\alpha_1}}{u^{\alpha_2}} \left(\frac{\Delta T}{T_g}\right)^{\alpha_3} D_0^{\alpha_4}. \tag{1}$$

Table 1 gives the exponents for the plume and wind parameters as derived by various workers; clearly, the wind speed has the largest effect, and this appears in the expressions for Δh_t given by various workers with powers ranging from the first to third. Further, the temperature difference appears with powers ranging from 0.25 to 1, while the stack diameter appears with powers ranging from 0.5 to 2, and so on.

These differences in the exponents for the thermal rise are due to the different assumptions employed in determining Δh_t, in conjunction with the lack of experimental data on the effects of the various factors on the rise. These are due in part to the difficulty in examining thermal rise on models under laboratory conditions. In fact, if tracers simulating a plume are injected into a wind tunnel as a turbulent flow perpendicular to the direction of motion of the wind (as is the actual situation), then there is rapid mixing of the two fluxes during the dynamic-rise phase, and so the temperature of the hot plume falls rapidly, so it is extremely difficult to examine the effects of the Archimedean force on such models, and it is even more difficult to examine the effects of other factors on plume shape and thermal rise. For example, measurements [7] indicate that a change in plume temperature from 100 to 200°C causes the path z_0/D_0 to alter by only 4.1% for $x/D_0 = 4$ or by 11% for $x/D_0 = 8$, which of itself is very slight. We are not aware of any measurements in wind tunnels on thermal rise or plumes of high heat content, particularly for large distances from the point of injection into an incident flow ($x/D_0 > 10$).

The Moscow Power Institute has performed experiments in wind tunnels and on existing power stations in order to determine the effects of the various factors on the hydrodynamic and thermal components of plume rise above stacks.

Physical simulation was employed with models; the wind tunnel had a closed working section.

This tunnel had a cross section of 650 × 650 mm ($d_e = 732$) and a length of 2000 mm [8].

An isotropic isothermal flow was used in examining plume rise; the following factors justify this simulation: the abundant available evidence [9] indicates a logarithmic distribution for the wind speed near the ground, while the speed remains essentially constant above the boundary layer, and the turbulent-diffusion coefficient increases with height but remains virtually constant and equal ($k_y = k_z$) outside the boundary layer (above ≈100 m).

Therefore, plume rise in the atmosphere, particularly over a high stack, may be simulated as the motion of a heated jet in a transverse isotropic turbulent flow with a uniform velocity profile. Isothermal conditions in the flow simplify the problem but do not detract from the generality of the conclusions on plume behavior.

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TABLE 1. Exponents for Plume and Wind Parameters and Coefficient of Proportionality for Thermal Plume Rise

Source	B	α_1	α_2	α_3	α_4
Lukas et al. [3]	1	0,5	1	0,5	0,5
Moses and Carson [4]	9,4–36,4*	0,5	1	0,5	1
Kholland [5]	2,845	1	1	1	2
Bosanquet [2]	15	1	3	1	2
Glavnoi Geophysical Laboratory data [1]	17,9	1	3	1	2
Morton [6]	0,5	0,25	0	0,25	0,5
Stumke [3]	8,3	1	1	1	2

*The values vary with the state of the atmosphere.

Definitive criteria can be derived by considering the equilibrium conditions for the forces on a component of a hot jet in a turbulent wind. In our case, these are the Reynolds and Archimedes numbers together with the hydrodynamic parameter $I = \rho_0 w_0^2/\rho_u u^2$.

Finally, the model may be fitted to the actual circumstances provided that the incident wind has the same structure, i.e., the same turbulence characteristics or else $K_a = b_0/u^2$, where b_0 is the characteristic turbulent energy of the incident wind. We do not envisage complex similarity and do not specify identical scales for the eddies (clearly, large eddies cannot occur within the stack), and therefore we utilize the turbulence characteristic $\varepsilon = \sqrt{\overline{u'^2}}/u$ to characterize the flow structure, where u' and u are respectively the wind-speed fluctuation and the mean speed. It is found [10] that the range in the above parameter is 0.08–0.27 for the ground-level layer of air under the conditions most characteristic of plumes from stacks. We produced flows with this structure by the use of immobile turbulizing grids with various geometrical showed that reasonably homogeneous isotropic turbulent flows with the appropriate turbulence can be produced by such grids [11]. The hydrodynamic and thermal components of plume rise were examined independently on the models. For this purpose we examined the hydrodynamic component on a jet injected perpendicular to the wind. On the other hand, the thermal component was examined in pure form by a novel technique, in which the thermal rise was examined. The stack model was oriented horizontally along the incident flow, which produced a flow coaxial with the incident wind having the same speed and the same turbulence, with the latter provided by a suitable turbulizer fitted into the stack.

Figures 1 and 2 show the general results on the hydrodynamic and thermal components of the rise.

Figure 1a shows that the plumes develop identically as functions of the hydrodynamic parameter, although the rises themselves vary; the end of the hydrodynamic rise may be taken as the point when the inclination of the tangent to the plume is $\beta = 10°$, in which case the relative rise $\Delta h_h/D_0$ is a linear function of the hydrodynamic parameter in a log–log plot (Fig. 1b).

$$\frac{\Delta h_h}{D_0} = 1.1 I^{0.6} \quad \text{or} \quad \Delta h_h = 1.1 D_0 \left(\frac{w_0}{u}\right)^{1.2} \left(\frac{T_a}{T_g}\right)^{0.6}. \tag{2}$$

The following ranges in the input parameters were used for the thermal rise: u = 0.7–3 m/sec, ΔT = 70–200°K, ε = 0.05–0.17, D_0 = 0.03–0.08 m; Fig. 2a shows the plume shapes (Fig. 2b shows the same data in dimensionless form, while Fig. 2c shows the generalized relationship for the rise in terms of the thermal-rise criterion K_{tr}). These results on the effects of wind and plume parameters on the thermal component define the exponents and the coefficient of proportionality in (1), and the latter then becomes

$$\Delta h_t = \frac{g D_0^2 w}{4 c \varepsilon u^3} \frac{\Delta T}{T_g} \frac{1}{\operatorname{tg} \beta} = B D_0 K_{tr}, \tag{3}$$

where $B = 1/4c\tan\beta$ is the coefficient of proportionality, while $K_{tr} = g D_0 w_0 \Delta T/\varepsilon u^3 T_g$ is the thermal-rise number, which differs from the Archimedes number in that the wind speed appears to the third power in the thermal rise, while the plume speed at the exit from the stack is raised to the first power, as does the turbulence for either flow; c is the resistance coefficient for a conical element moving upwards with speed w (experiment gives c = 1.6), while β is the inclination of the tangent to the plume at the point where the rise terminates.

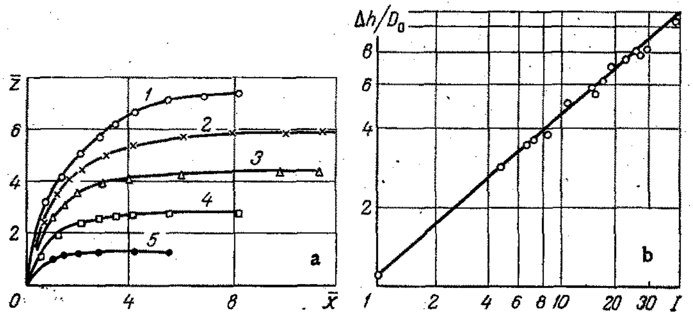

Fig. 1. Observed hydrodynamic-rise curves for plumes in turbulent incident winds (a) [1) I = 22.4; 2) 15; 3) 8.7; 4) 4.6; 5) 1.0]; relative plume rise as a function of the hydrodynamic parameter (b) ($\Delta h/d_0 = 1.10\, I^{0.6}$; $I = \rho_0 W_0^2/\rho u^2$).

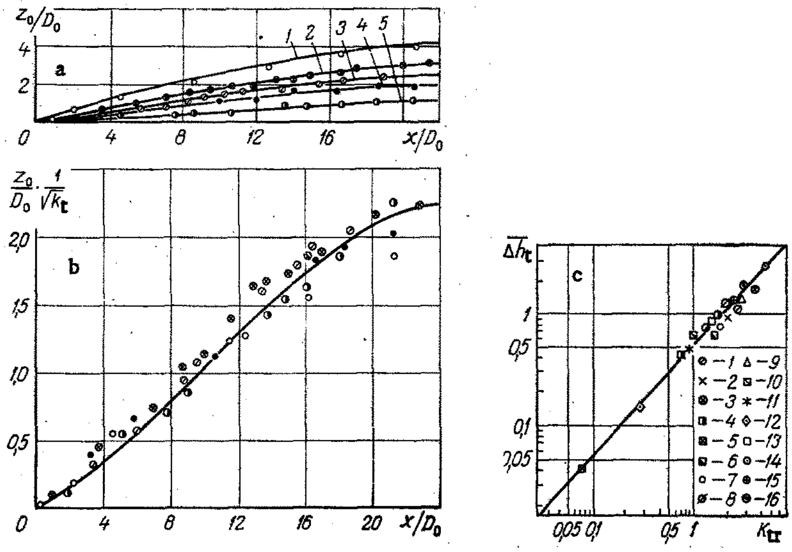

Fig. 2. Thermal plume rise in a turbulent incident wind (a) [1) D_0 = 0.050 m; T_g = 441°K; ΔT = 140°K; u = 1.0 m/sec; ε = 0.07; 2) respectively 0.042; 418; 120; 1.0; 0.07; 3) 0. 030; 418; 120; 1.0; 0.07; 4) 0.050; 418; 120; 1.0; 0.17; 5) 0.050; 441; 3.0; 0.07]; thermal plume rise in generalized form (b); thermal rise in relation to the thermal-rise number (c) [1) ε = 0.07; u = 1.0 m/sec; ΔT = 120°K; T_g = 418°K; D_0 = 30 m; 2) respectively 0.07; 1.0; 120; 418; 42; 3) 0.07; 0.6; 60; 355; 50; 4) 0.07; 1.3; 70; 371; 50; 5) 0.07; 5.0; 110; 408; 50; 6) 0.07; 1.3; 110; 411; 50; 7) 0.1; 1.0; 120; 418; 50; 8) 0.07; 1.0; 120; 418; 50; 9) 0;05; 1.0; 120; 418; 50; 10) 0.14; 1.0; 120; 418; 50; 11) 0.17; 1.0; 120; 418; 50; 12) 0.07; 3.0; 140; 441; 50; 13) 0.07; 1.3; 140; 441; 50; 14) 0.07; 0.7; 140; 441; 50; 15) 0.07; 1.0; 140; 440; 50; 16) 0.07; 1.0; 120; 418, 80].

These wind-tunnel data therefore define the shape of the plume in relation to the hydrodynamic and thermal effects; it is therefore possible to determine the rise. Over the main section of the hydrodynamic rise [7], and particularly over the thermal rise section, the motion of the plume is described by a parabolic formula [8]. Undoubtedly, this is influenced to some extent by the approximations we have employed (isotropic isothermal flow), and this must set limits to the applicability of the working formulas, in particular, for the hydrodynamic component. Figure 1 shows that a power-law relationship applies for the hydrodynamic rise only at the start (at distances of 6-8 times the stack diameter along the x axis). The curve then becomes very nearly a horizontal straight line, so the inclination of the tangent taken at the point of termination plays virtually no part.

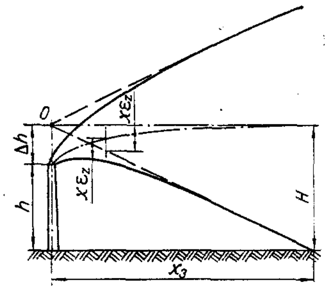

Fig. 3. Theoretical model for calculating plume rise.

Therefore, the overall rise can be defined by simple combination of the hydrodynamic and thermal components, so we have the following formula derived from the model experiments:

$$\Delta h = \Delta h_{\mathrm{h}} + \Delta h_{\mathrm{t}} = 1.2 \frac{w_0 D_0}{u} + 0.15 \frac{g w_0 D_0^2 \Delta T}{T_{\mathrm{g}} u^3 \varepsilon_y \operatorname{tg} \beta}, \tag{4}$$

where the coefficient of proportionality 1.2 has been derived from (2) for the conditions existing in thermal power stations.

Under real circumstances, the two components occur together, so any division of the rise into two components is nominal; however, the model data and the plume shapes representing the two components can be used in an appropriate correction for the difference between real processes and the laboratory ones in order to write the actual plume shape as

$$z = Kx^n,$$

where

$$K = \sqrt{K_{\mathrm{h}} \frac{w_0 D_0}{u} + K_{\mathrm{t}} \frac{g w_0 D_0^2}{u^3 \varepsilon_y} \frac{\Delta T}{T_{\mathrm{g}}}}. \tag{5}$$

This implies that the plume shape derived from trials under actual conditions may reasonably be plotted in semilogarithmic form; in that case, the exponent can be deduced from any two points on the plume, while K may be deduced from the coordinates of any point.

In 1974-7, the Moscow Power Institute collaborated with the Novosibirsk Applied-Geodesy Institute and the Ukrainian Hydrometeorology Research Institute in measurements on plumes above stacks designed to check the formulas derived from model tests and to establish the limits to plume rise, particularly as influenced by actual atmospheric parameters. These measurements were made on the Kashir, Kostroma, and Zaporozhe power stations during the production of plumes differing in heat content, speed, and height of emergence into the atmosphere. The heights of the stacks varied from 150 to 320 m. The plumes were recorded with a theodolite by determination of the horizontal and vertical angles on sighting on the boundary of the plume (10-15 points along the plume). The theodolite measurements were accompanied by aircraft sampling, photographic recording of the plume, and stereophotogrammatic recording, so a computer could be employed to compute any plume shape with high accuracy [12]. These various means of recording were employed to provide scope for comparison and for definition of the cheapest and simplest method of performing the observations.

These data were used in plotting over 500 plumes in logarithmic form for observations relating to distances ranging from 300-700 m up to 5-6 km from the stacks. About 85% of the plumes fitted closely to straight lines, while in about 150 there was a kink, which in most cases was at the end of the path, and which might be due to nonuniformity in the wind-speed profile or (if the atmospheric turbulence is of considerable scale and extent) to breakup into individual clouds, in addition to any effect from heat transfer between the plume and the atmosphere.

The results indicate that (5) describes the plume shape; n varies over the range 0.4-0.7. Many of the values of n lie in the range 0.45-0.65, and the average value is 0.52, which is virtually the same as the theoretical value used in deriving (2)-(4).

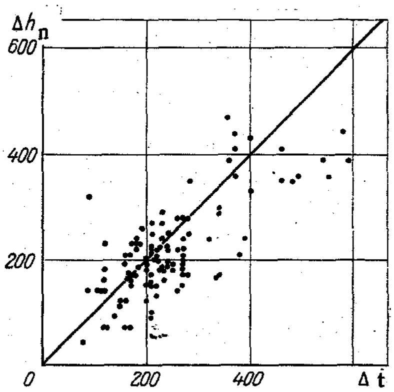

Fig. 4. Comparison of observed and calculated plume rises.

If the plume shape is known, the rise is defined by the point at which the angle between the tangent and the horizontal axis attains some comparatively small specified value. So far, uncertainty persists in defining the approach to the horizontal, i.e., the value for this angle. Also, it is not always possible to define the rise itself very precisely. For example, in some papers one finds the rise taken as the height of the plume at which the inclination of the tangent is $\beta = 10°$, whereas in other papers the rise is taken as the height at a specified distance from the stack or at a specified time, and so on. See [9] for a survey of these studies.

The limiting rise may be examined by replacing the actual path by the theoretical one; in that case, a stack of height h is replaced by a point source that emits gases with no initial thermal or inertial momentum, with this source placed at a height Δh above the stack such that the ground-level concentration pattern is approximately the observed one (Fig. 3). This shows that Δh should be taken at the height of the median line in the plume at the point where the lower boundary reaches the ground. The equation for the lower boundary of the plume can be derived from (5) in the following form for an isotropic incident wind:

$$z = h + Kx^n - \varepsilon_z x, \tag{6}$$

where ε_z is the atmospheric turbulence in the vertical plane. Then $z = 0$ gives us the point of contact of the lower boundary with the ground.

In the case of a parabola, i.e., $n = 0.5$,

$$x_z = \frac{K^2 + 2h\varepsilon_z + K\sqrt{K^2 + 4h\varepsilon_z}}{2\varepsilon_z^2}. \tag{7}$$

The recordings show that $n = 0.5$ (on average $n = 0.4\text{–}0.65$) is characteristic of the plume for lengths up to $x/D_0 = 120$; there is a tendency for n to fall for longer sections. The observed plumes allow one to simplify the calculations by using $n = 0.5$ out to $x/D_0 = 120$, with $n = 0.35$ thereafter. In that case, the point of contact of the lower bound with the ground for $x/D_0 > 120$ is defined by successive approximation.

Once the point of contact with the ground has been defined, it is simple to determine the rise and to define the pollutants concentration pattern. The measurements gave the values for K_h and K_t in (5) as 0.42 and 0.3 respectively.

Figure 4 compares the natural rise (Δh_n) and the theoretical values given by (5) for $x/D_0 = 120$; clearly, the two are fairly closely correlated, and the correlation coefficient is $r_{xy} = 0.8$.

From this we conclude that the theoretical Δh are in satisfactory agreement with actual observations.

NOTATION

B	is the coefficient of proportionality;
w_0	is the speed at exit, m/sec;
u	is the mean wind speed, m/sec;
ΔT	is the difference between mean temperatures of flue gas and wind, °K;
T_g	is the gas temperature at exit, °K [in the expressions given by Bosanquet and Morton and by the Main Geophysical Observatory, T_a (the air temperature at the wind vane) is used instead of T_g];

D_0	is the orifice diameter, m;
α_1-α_4	are the exponents;
ρ_0, w_0	are the density and velocity of jet;
ρ_u	is the density of incident flow;
ε, ε_x, ε_y	are the turbulence of isotropic flow and the same in the vertical and horizontal planes;
z, x	are the coordinates;
K_h, K_t	are the coefficients of proportionality of hydrodynamic and thermal components of plume rise.

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